OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4755

Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4755
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Wednesday 19 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Turn over

Section A (36 marks)

1 Find the values of P, Q, R and S in the identity $3x^3 + 18x^2 + Px + 31 \equiv Q(x+R)^3 + S$. **[5]**

2 You are given that $\mathbf{M} = \begin{pmatrix} 4 & 0 \\ -1 & 3 \end{pmatrix}$.

(i) The transformation associated with $\mathbf{M}$ is applied to a figure of area 3 square units. Find the area of the transformed figure. **[2]**

(ii) Find $\mathbf{M}^{-1}$ and $\det \mathbf{M}^{-1}$. **[3]**

(iii) Explain the significance of $\det \mathbf{M} \times \det \mathbf{M}^{-1}$ in terms of transformations. **[2]**

3 The roots of the cubic equation $x^3 - 4x^2 + 8x + 3 = 0$ are α, β and γ.

Find a cubic equation whose roots are $2\alpha - 1$, $2\beta - 1$ and $2\gamma - 1$. **[7]**

4 Represent on an Argand diagram the region defined by $2 < |z - (3 + 2\mathrm{j})| \leqslant 3$. **[6]**

5 Use standard series formulae to show that $\sum_{r=1}^{n} r^2(3 - 4r) = \frac{1}{2}n(n+1)(1 - 2n^2)$. **[5]**

6 A sequence is defined by $u_1 = 5$ and $u_{n+1} = u_n + 2^{n+1}$. Prove by induction that $u_n = 2^{n+1} + 1$. **[6]**

Section B (36 marks)

7 Fig. 7 shows part of the curve with equation $y = \dfrac{x+5}{(2x-5)(3x+8)}$.

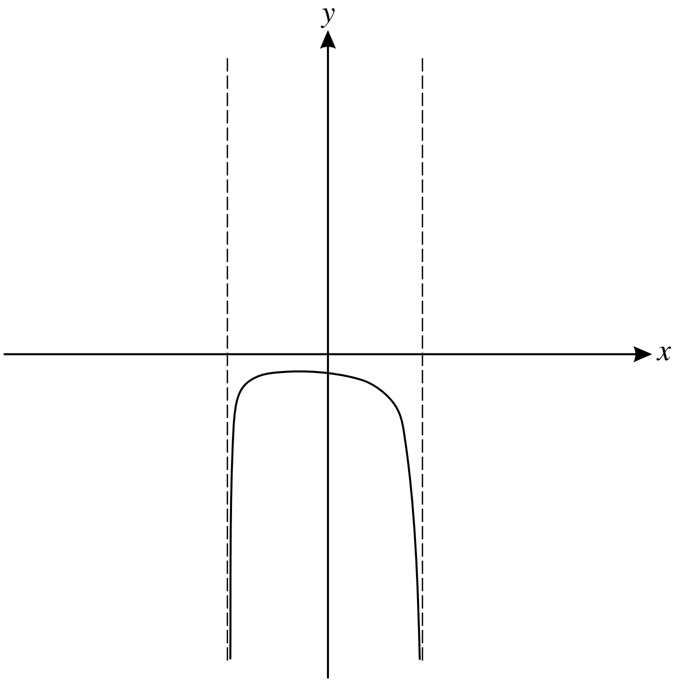

Fig. 7

(i) Write down the coordinates of the two points where the curve crosses the axes. **[2]**

(ii) Write down the equations of the two vertical asymptotes and the one horizontal asymptote. **[3]**

(iii) Determine how the curve approaches the horizontal asymptote for large positive and large negative values of x. **[3]**

(iv) On the copy of Fig. 7, sketch the rest of the curve. **[2]**

(v) Solve the inequality $\dfrac{x+5}{(2x-5)(3x+8)} < 0$. **[2]**

8 The function $f(z) = z^4 - z^3 + az^2 + bz + c$ has real coefficients. The equation $f(z) = 0$ has roots α, β, γ and δ where $\alpha = 1$ and $\beta = 1 + j$.

(i) Write down the other complex root and explain why the equation must have a second real root. **[2]**

(ii) Write down the value of $\alpha + \beta + \gamma + \delta$ and find the second real root. **[3]**

(iii) Find the values of a, b and c. **[5]**

(iv) Write down $f(-z)$ and the roots of $f(-z) = 0$. **[2]**

9 You are given that $\mathbf{A} = \begin{pmatrix} -2 & 1 & -5 \\ 3 & a & 1 \\ 1 & -1 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2a+1 & 3 & 1+5a \\ -5 & 1 & -13 \\ -3-a & -1 & -2a-3 \end{pmatrix}$.

(i) Show that $\mathbf{AB} = (8+a)\mathbf{I}$. **[3]**

(ii) State the value of a for which $\mathbf{A}^{-1}$ does not exist. Write down $\mathbf{A}^{-1}$ in terms of a, when $\mathbf{A}^{-1}$ exists. **[3]**

(iii) Use $\mathbf{A}^{-1}$ to solve the following simultaneous equations. **[5]**

$$\begin{aligned} -2x + y - 5z &= -55 \\ 3x + 4y + z &= -9 \\ x - y + 2z &= 26 \end{aligned}$$

(iv) What can you say about the solutions of the following simultaneous equations? **[1]**

$$\begin{aligned} -2x + y - 5z &= p \\ 3x - 8y + z &= q \\ x - y + 2z &= r \end{aligned}$$

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MATHEMATICS (MEI) 4755

Further Concepts for Advanced Mathematics (FP1)

PRINTED ANSWER BOOK

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- MEI Examination Formulae and Tables (MF2)

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Wednesday 19 January 2011
Afternoon

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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Turn over

Section A (36 marks)

1

2 (i)	
2 (ii)	
2 (iii)	

3

4	

5

5	**(continued)**

6

6	**(continued)**

Section B (36 marks)

7 (i)	
7 (ii)	
7 (iii)	

7 (iv)

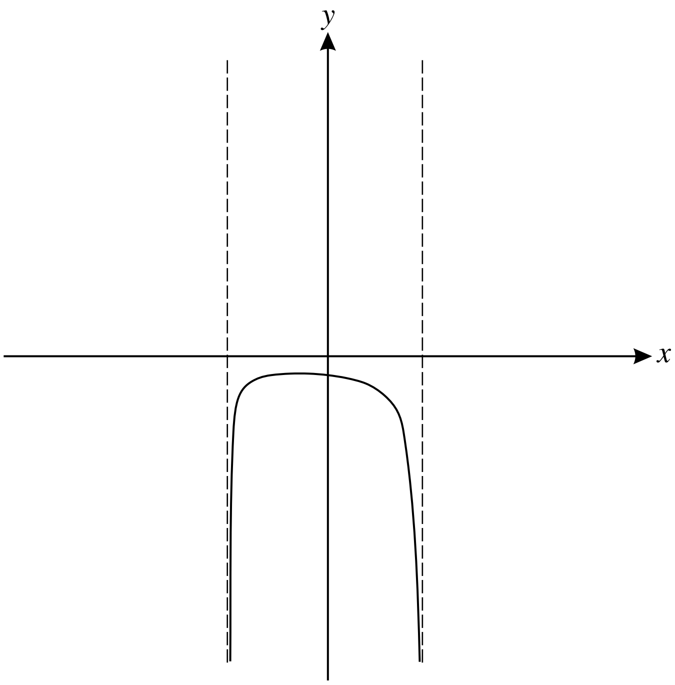


Fig. 7

7 (v)

8 (i)	
8 (ii)	
8 (iii)	

8 (iii)	(continued)
8 (iv)	

9 (i)	
9 (ii)	

9 (iii)	

9 (iv)

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4755:** Further Concepts for Advanced Mathematics

Mark Scheme for January 2011

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of pupils of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

Marking instructions for GCE Mathematics (MEI): Pure strand

1. You are advised to work through the paper yourself first. Ensure you familiarise yourself with the mark scheme before you tackle the practice scripts.

2. You will be required to mark ten practice scripts. This will help you to understand the mark scheme and will not be used to assess the quality of your marking. Mark the scripts yourself first, using the annotations. Turn on the comments box and make sure you understand the comments. You must also look at the definitive marks to check your marking. If you are unsure why the marks for the practice scripts have been awarded in the way they have, please contact your Team Leader.

3. When you are confident with the mark scheme, mark the ten standardisation scripts. Your Team Leader will give you feedback on these scripts and approve you for marking. (If your marking is not of an acceptable standard your Team Leader will give you advice and you will be required to do further work. You will only be approved for marking if your Team Leader is confident that you will be able to mark candidate scripts to an acceptable standard.)

4. Mark strictly to the mark scheme. If in doubt, consult your Team Leader using the messaging system within *scoris*, by email or by telephone. Your Team Leader will be monitoring your marking and giving you feedback throughout the marking period.

   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

   Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

5. The following types of marks are available.

   M
   A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

   A
   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

6. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

7. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

   Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

8. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

9. **Rules for crossed out and/or replaced work**

   If work is crossed out and not replaced, examiners should mark the crossed out work if it is legible.

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If two or more attempts are made at a question, and just one is not crossed out, examiners should ignore the crossed out work and mark the work that is not crossed out.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

10. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

11. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

12. For answers scoring no marks, you must either award NR (no response) or 0, as follows:

Award NR (no response) if:

- Nothing is written at all in the answer space
- There is a comment which does not in any way relate to the question being asked ("can't do", "don't know", etc.)
- There is any sort of mark that is not an attempt at the question (a dash, a question mark, etc.)

The hash key [#] on your keyboard will enter NR.

Award 0 if:

- There is an attempt that earns no credit. This could, for example, include the candidate copying all or some of the question, or any working that does not earn any marks, whether crossed out or not.

13. The following abbreviations may be used in this mark scheme.

M1	method mark (M2, etc, is also used)
A1	accuracy mark
B1	independent mark
E1	mark for explaining
U1	mark for correct units
G1	mark for a correct feature on a graph
M1 dep*	method mark dependent on a previous mark, indicated by *
cao	correct answer only
ft	follow through
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
sc	special case
soi	seen or implied
www	without wrong working

14. Annotating scripts. The following annotations are available:

✓and ✗

BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working (after correct answer obtained)
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0,1
SC	Special case
^	Omission sign
MR	Misread

Highlighting is also available to highlight any particular points on a script.

15. The comments box will be used by the Principal Examiner to explain his or her marking of the practice scripts for your information. Please refer to these comments when checking your practice scripts.

Please do not type in the comments box yourself. Any questions or comments you have for your Team Leader should be communicated by the *scoris* messaging system, e-mail or by telephone.

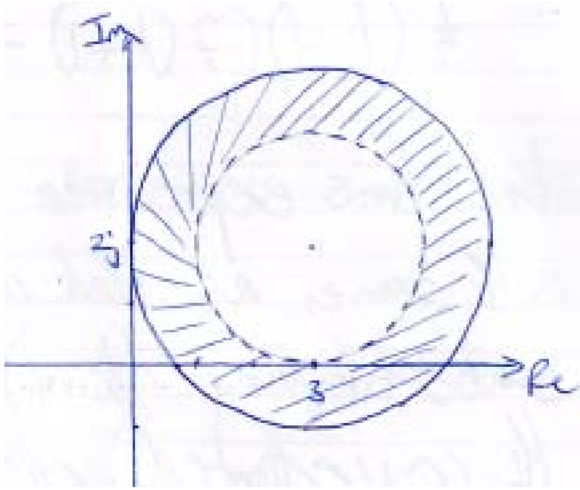
16. Write a brief report on the performance of the candidates. Your Team Leader will tell you when this is required. The Assistant Examiner's Report Form (AERF) can be found on the Cambridge Assessment Support Portal. This should contain notes on particular strengths displayed, as well as common errors or weaknesses. Constructive criticisms of the question paper/mark scheme are also appreciated.

17. Link Additional Objects with work relating to a question to those questions (a chain link appears by the relevant question number) – see scoris assessor Quick Reference Guide page 19-20 for instructions as to how to do this – this guide is on the Cambridge Assessment Support Portal and new users may like to download it with a shortcut on your desktop so you can open it easily! For AOs containing just formulae or rough working not attributed to a question, tick at the top to indicate seen but not linked. When you submit the script, *scoris* asks you to confirm that you have looked at all the additional objects. Please ensure that you have checked all Additional Objects thoroughly.

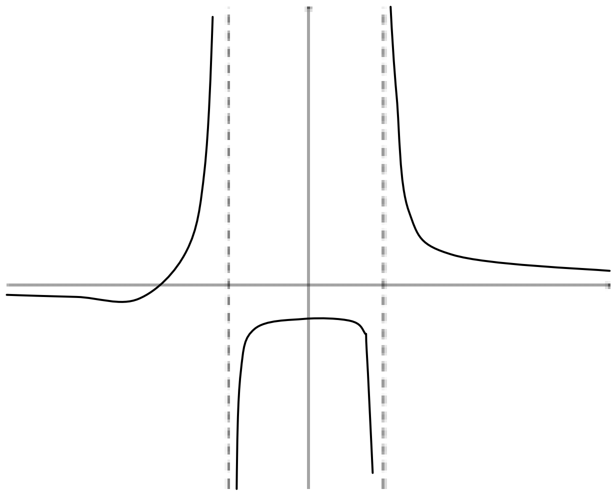
18. The schedule of dates for the marking of this paper is displayed under 'OCR Subject Specific Details' on the Cambridge Assessment Support Portal. It is vitally important that you meet these requirements. If you experience problems that mean you may not be able to meet the deadline then you must contact your Team Leader without delay.

Qu	Answer	Mark	Comment
1	$3x^3+18x^2+Px+31\equiv Q(x+R)^3+S$ $Q=3$ $3x^3+18x^2+Px+31\equiv 3x^3+9Rx^2+9R^2x+3R^3+S$ $R=2,\ P=36,\ S=7$	B1 M1 A3 **[5]**	Q = 3 anywhere Attempt to expand and compare at least another coefficient, or other valid method One mark for each correct constant cao
2(i)	$\det\mathbf{M}=4\times3-(-1)\times0$ Area $=12\times3=36$ square units	M1 A1 **[2]**	oe www
2(ii)	$\mathbf{M}^{-1}=\frac{1}{12}\begin{pmatrix}3&0\\1&4\end{pmatrix}$ $\det\mathbf{M}^{-1}=\frac{1}{12}$	M1 A1 B1 **[3]**	division by their $\det\mathbf{M}$ cao condone decimals 3sf or better cao condone decimal 3sf or better
2(iii)	$\det\mathbf{M}\times\det\mathbf{M}^{-1}=12\times\frac{1}{12}=1$ The inverse 'undoes' the transformation, so the composite of **M** and its inverse must leave a shape unchanged, meaning the area scale factor of the composite transformation must be 1 and so the determinant is 1.	B1 E1 **[2]**	Seen or implied Any valid explanation involving transformations and unchanged area

3	$\omega = 2x - 1 \Rightarrow x = \dfrac{\omega+1}{2}$ $\left(\dfrac{\omega+1}{2}\right)^3 - 4\left(\dfrac{\omega+1}{2}\right)^2 + 8\left(\dfrac{\omega+1}{2}\right) + 3 = 0$ $\Rightarrow \frac{1}{8}\left(\omega^3 + 3\omega^2 + 3\omega + 1\right) - \left(\omega^2 + 2\omega + 1\right)$ $+4\left(\omega+1\right) + 3 = 0$ $\Rightarrow \omega^3 - 5\omega^2 + 19\omega + 49 = 0$	M1 A1 M1 M1 A2 A1 **[7]**	Using a substitution Correct Substitute into cubic Attempting to expand cubic and quadratic LHS oe, -1 each error Correct equation
3	OR $\alpha + \beta + \gamma = 4$ $\alpha\beta + \alpha\gamma + \beta\gamma = 8$ $\alpha\beta\gamma = -3$ Let new roots be *k*, *l*, *m* then $k + l + m = 2\left(\alpha + \beta + \gamma\right) - 3 = 5 = \dfrac{-B}{A}$ $kl + km + lm = 4\left(\alpha\beta + \alpha\gamma + \beta\gamma\right)$ $-4\left(\alpha + \beta + \gamma\right) + 3 = 19 = \dfrac{C}{A}$ $klm = 8\alpha\beta\gamma - 4\left(\alpha\beta + \alpha\gamma + \beta\gamma\right)$ $+2\left(\alpha + \beta + \gamma\right) - 1 = -49 = \dfrac{-D}{A}$ $\Rightarrow \omega^3 - 5\omega^2 + 19\omega + 49 = 0$	M1 A1 M1 A1 A1 A1 A1 **[7]**	Attempt to find $\Sigma\alpha$ $\Sigma\alpha\beta$ $\alpha\beta\gamma$ All correct Attempt to use root relationships to find at least two of Σk Σkl klm Quadratic coefficient Linear coefficient Constant term Correct equation

4		B1 B1 B1 B1 B1 B1 **[6]**	Circle Centre 3 + 2j Radius = 2 or 3, consistent with their centre Both circles correct cao Correct boundaries indicated, inner excluded, outer included (f t concentric circles) Region between concentric circles indicated as solution SC -1 if axes incorrect
5	$\sum_{r=1}^{n} r^2(3-4r) = 3\sum_{r=1}^{n} r^2 - 4\sum_{r=1}^{n} r^3$ $= \frac{3}{6}n(n+1)(2n+1) - \frac{4}{4}n^2(n+1)^2$ $= \frac{1}{2}n(n+1)[(2n+1)-2n(n+1)]$ $= \frac{1}{2}n(n+1)(1-2n^2)$	M1 M1 A1 M1 A1 **[5]**	Separate into two sums involving r^2 and r^3, may be implied Appropriate use of at least one standard result Both terms correct Attempt to factorise using both n and $n + 1$ Complete, convincing argument

6	When n = 1, $2^{1+1}+1=5$, so true for n = 1	B1	
	Assume $u_k = 2^{k+1}+1$	E1	Assuming true for k
	$\Rightarrow u_{k+1} = 2^{k+1}+1+2^{k+1}$	M1	Using this u_k to find u_{k+1}
	$= 2\times 2^{k+1}+1$		
	$= 2^{k+2}+1$	A1	Correct simplification
	$= 2^{(k+1)+1}+1$		
	But this is the given result with k + 1 replacing k. Therefore if it is true for k it is also true for k + 1.	E1	Dependent on A1 and previous E1
	Since it is true for n = 1, it is true for all positive integers.	E1 **[6]**	Dependent on B1 and previous E1

7(i)	$\left(0, \frac{-1}{8}\right)$, $(-5, 0)$	B1 B1 **[2]**	One mark for each point SC1 for x = -5, y = -1/8
7(ii)	$x = \frac{5}{2}, x = \frac{-8}{3}, y = 0$	B1 B1 B1 **[3]**	One mark for each equation
7(iii)	Large positive x, $y \to 0^+$ (e.g. consider x = 100) Large negative x, $y \to 0^-$ (e.g. consider x = -100)	B1 B1 M1 **[3]**	 Evidence of a valid method
7(iv)		B1 B1 **[2]**	RH branch correct LH branch correct
7(v)	$x < -5$ or $\frac{-8}{3} < x < \frac{5}{2}$	B1 B1 **[2]**	cao cao

8(i)	$\delta = 1 - \mathrm{j}$ There must be a second real root because complex roots occur in conjugate pairs.	B1 E1 **[2]**	
8(ii)	$\alpha + \beta + \gamma + \delta = 1$ $\alpha + \beta + \gamma + \delta = 1 \Rightarrow 1 + (1 + j) + \gamma + (1 - j) = 1$ $\Rightarrow \gamma = -2$	B1 M1 A1 **[3]**	 cao
8(iii)	$(z-1)(z+2)(z-(1+j))(z-(1-j))$ $= (z^2 + z - 2)(z^2 - 2z + 2)$ $= z^4 - 2z^3 + 2z^2 + z^3 - 2z^2 + 2z - 2z^2 + 4z - 4$ $= z^4 - z^3 - 2z^2 + 6z - 4$ $\Rightarrow a = -2,\ b = 6,\ c = -4$	B1 M1 A3 **[5]**	Correct factors from their roots Attempt to expand using all 4 factors One for each of *a*, *b* and *c*
	OR		
	$\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = a = -2$ $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -b = -6 \Rightarrow b = 6$ $\alpha\beta\gamma\delta = c = -4$	M2 A1 A1 B1 **[5]**	Use of root relationships attempted, M2 evidence of all 3, M1 for evidence of 2 OR substitution to get three equations and solving *a* = -2 cao *b* = 6 cao *c* = -4 (SC f t on their 2nd real root)
8(iv)	$\mathrm{f}(-z) = z^4 + z^3 - 2z^2 - 6z - 4$ Roots of $\mathrm{f}(-z) = 0$ are $-1,\ 2,\ -1 + \mathrm{j}$ and $-1 - \mathrm{j}$	B1 B1 **[2]**	f t on their *a*, *b*, *c*, simplified For all four roots, cao

9(i)	$\mathbf{AB}=\begin{pmatrix}-2 & 1 & -5\\ 3 & a & 1\\ 1 & -1 & 2\end{pmatrix}\begin{pmatrix}2a+1 & 3 & 1+5a\\ -5 & 1 & -13\\ -3-a & -1 & -2a-3\end{pmatrix}$		
	$=\begin{pmatrix}-4a-2-5+15+5a & 0 & 0\\ 0 & 9+a-1 & 0\\ 0 & 0 & 1+5a+13-4a-6\end{pmatrix}$	M1	Attempt to find **AB** with some justification of at least two leading diagonal terms and any other
	$=\begin{pmatrix}8+a & 0 & 0\\ 0 & 8+a & 0\\ 0 & 0 & 8+a\end{pmatrix}$	A1	Correct
	$=(8+a)\mathbf{I}$	B1	Relating correct diagonal matrix to **I**
		[3]	
9(ii)	$\mathbf{A}^{-1}$ does not exist for $a = -8$	B1	
	$\mathbf{A}^{-1}=\frac{1}{8+a}\begin{pmatrix}2a+1 & 3 & 1+5a\\ -5 & 1 & -13\\ -3-a & -1 & -2a-3\end{pmatrix}$	M1 A1	$k\mathbf{B}$, k not equal to 1 Correct $\mathbf{A}^{-1}$ as shown
		[3]	
9(iii)	$\mathbf{A}^{-1}=\frac{1}{12}\begin{pmatrix}9 & 3 & 21\\ -5 & 1 & -13\\ -7 & -1 & -11\end{pmatrix}$	B1	
	$\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\frac{1}{12}\begin{pmatrix}9 & 3 & 21\\ -5 & 1 & -13\\ -7 & -1 & -11\end{pmatrix}\begin{pmatrix}-55\\ -9\\ 26\end{pmatrix}=\begin{pmatrix}2\\ -6\\ 9\end{pmatrix}$	M1 A3 **[5]**	Correct use of their $\mathbf{A}^{-1}$ x, y and z cao, -1 each error
9(iv)	There is no unique solution.	B1 **[1]**	

4755 Further Concepts for Advanced Mathematics

General Comments

Most candidates found the paper accessible and were able to demonstrate knowledge and ability in dealing with mathematical expressions. The overall standard was high, with most responses showing work with good mathematical presentation. Candidates appeared to be scoring more highly in Section A than in Section B on this occasion. It may be that quite a few candidates found themselves working against the clock towards the end of the paper, where there were some instances of no response within parts of both the final questions 8 and 9.

Comments on Individual Questions

1) Mostly well done. It was surprising that many candidates were unable to write down the expansion of $(x+R)^3$, but needed to expand and multiply out $(x+R)^2\ (x+R)$ Not all did this successfully. Another common error was to fail to multiply the cubic expansion by Q completely, usually resulting in the wrong expression for S, and sometimes for P as well.

2) (i) This was very well done, with most candidates choosing to use the determinant as the appropriate area scale factor. A few tried to transform a particular shape and work out the area of the new shape. This lacked generality and was not carefully explained.
(ii) Well done by the majority of candidates. There was evidence of confusion in terminology here, where some gave a matrix for the determinant det $\mathbf{M}^{-1}$. Others gave 12 as their answer, not being able to distinguish between det $\mathbf{M}$ and det $\mathbf{M}^{-1}$, and another fairly frequent error was to see 1/13 or 13 for the determinant, also 1 and 0.
(iii) Not many candidates achieved a coherent and concise comment on the value of det $\mathbf{M}$ x det $\mathbf{M}^{-1}$ which involved the idea of the area scale factor and its role in the transformations. An answer which referred only to matrices was insufficient.

3) By far the most popular route chosen through this question was by use of the root relationships $\sum\alpha$, $\sum\alpha\beta$ and $\alpha\beta\gamma$. Some candidates who successfully navigated through the corresponding expressions using the new roots forgot to give an equation at the end, as requested. Others stumbled in the algebra. The substitution method was not quite so popular but usually resulted in a more concise solution but which was still prone to error. The chief mistake was to forget the final term +3 when multiplying to eliminate the fractions. A few candidates used the wrong substitution, $2w - 1$ and $w/2 - 1$ were both seen.

4) A high proportion of candidates achieved full marks in this question. Very few failed to produce a circle or circles with at least one appropriate radius and with the correct centre. The nature of each boundary was not always clearly defined and in any event it is wise to give a key to define the included and excluded boundaries as there is no universally accepted convention on this.

5) This question was also successfully answered by most candidates. Only a few failed to begin with the separation into terms in $\sum r^2$ and $\sum r^3$. These were normally correct. Most candidates then saw the common factors of n and $(n + 1)$ and quickly showed the result. Some candidates expanded each term into a polynomial in n and then had to factorise again. The final step was not always convincing, as the answer was given.

6) The answers to this question were variable in quality. Where the candidate had thoroughly absorbed the recommended wording (as has been set out in many previously published mark schemes) there was complete success, but many missed the final two marks through not producing a satisfactory "If…then…." argument as they tried to use their own words. Most candidates stated that $2^{k+1} + 1 + 2^{k+1}$ was the same as $2^{k+2} + 1$, but this was most convincing when the intermediate step $2x2^{k+1} + 1$ was given as well. A very few candidates thought that they were dealing with the series $5 + 9 + \ldots + u_k$.

7) This was the most confidently answered of the questions in Section B.
(i) Only a few candidates neglected to give full co-ordinates for the two points.
(ii) Only a few candidates neglected to write unambiguously three equations, in full. Nearly all gave the correct horizontal asymptote, but $y = 1/6$ and $y = 1/x$ were both seen.
(iii) Nearly all candidates gave a clear indication of method, here. Where large numbers were chosen to evaluate the expression in x, it was more acceptable to see the end of the calculation. An algebraic solution needed careful explanation. Infinity should not be used as a number.
(iv) The examiners were looking for a carefully drawn sketch with unambiguous asymptotic approaches and a clear minimum shown in the region $x< -5$, $y<0$.
(v) Where the graph was essentially correct, the inequalities followed confidently, and there were not many candidates who wrongly used an inclusive inequality.

8) (i) Most candidates found the complex root and many gave good explanations of the reason for another real root, although others were less than coherent. Some of the best answers recalled the nature of the graph of the function.
(ii) Many earned full marks here. The common errors were to claim the sum of the roots to be -1 or, less frequently, zero.
(iii)The two popular routes to finding a, b and c were by means of the root relationships or by multiplying out the factors associated with the four known roots. The former method tested clear thinking owing to the unconventional allocation of *a, b,* and *c* in the original equation, but most candidates negotiated that successfully. There were some mistakes in the expansions in the second method. A few candidates found that one or other of the coefficients was not real, which should have given pause for thought.
(iv) This was another test of careful thinking. Not many candidates scored both marks in this section, forgetting either that only odd powers of z would change sign in $f(-z)$, or that $1 \pm j$ would become $-1 \pm j$. Numerical answers from (iii) were expected in $f(-z)$.

9) (i) Maybe because of constraints of time, but many candidates failed sufficiently to justify the terms in the matrix product **AB,** necessary as the answer was given**.** It was also expected that the factorisation of the resulting diagonal matrix should be explicit, for an easy mark.
(ii) This was usually well answered but some candidates were evidently uncertain about whether $(8 + a)$ or $1/(8 + a)$ should be used with **B**. A few candidates believed that they had already started the next part, and gave a numerical inverse matrix.
(iii) Mostly well done, but quite a few candidates wasted time by failing to realise that $a=4$ was needed, until the end of their hard work, and some failed to notice this at all. There were also other numbers used in particular $a = 0$.
(iv) Very few candidates gave an acceptable answer to this. Where it was attempted, many gave "No solutions", or "an infinity of solutions", but not many gave both. "No unique solution" was sufficient for the remaining mark.

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

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Head office
Telephone: 01223 552552
Facsimile: 01223 552553